Wave Coupling by Warped Normal Modes*

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Summary—**It has been shown by J. S. Cook that wave power may be transferred from one to another of two coupled waveguides through a variation of their phase constants. It is now clear that this is but one example of a new principle of coupling which is here called "normal mode warping." Wave power inserted at one end of a coupled waveguide system may be made to appear at the other end with any desired power distribution by gradual warping of the normal mode field patterns along the coupler. In general, both variation of the coupling coefficient and phase constants are required. Much wider bands are theoretically possible than with any other distributed type of coupler. This principle may be applied to dielectric waveguides, birefringent media, and waveguides containing ferrite, to obtain both reciprocal and nonreciprocal couplers.**

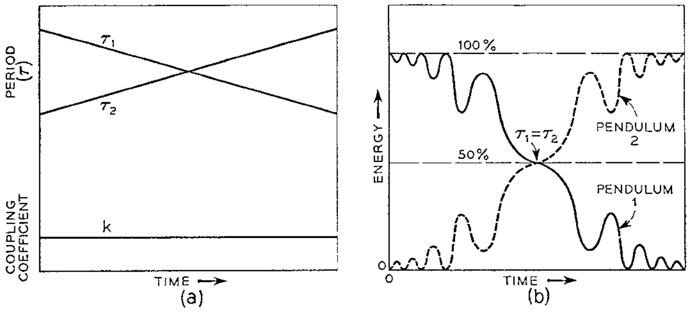
INTRODUCTION

IT IS NOW well known that complete transfer of power can be effected from one to another of two waveguides, provided there is distributed coupling between the waveguides and provided the phase velocities are equal.[1,2] A good illustrative analog is a pair of coupled pendulums having the same period. The periods correspond to the wavelengths in the waveguides; passage of time for the pendulums corresponds to distance along the waveguides; the energies in the pendulums at a particular time correspond to the wave powers in the two waveguides at a particular point along the waveguides. As energy is interchanged between pendulums with increasing time, power is interchanged between waveguides with distance.

To obtain a complete interchange of power for the waveguides a particular length is required which is determined by the coupling. As long as the coupling is constant, the power transfer should be independent of frequency. However, in practice, the coupling does vary with frequency, and hence the power division is a slowly varying function of frequency.

Recently a new and rather surprising method of transferring power has been described by J. S. Cook. He pointed out that if the phase constants of two waveguides are grossly "unequal" at one end, but are continuously varied so that they become "equal" in the middle of the coupling region and again grossly "unequal" in the opposite sense at the other end, complete power transfer should take place. Moreover, this transfer is independent of the size of the coupling coefficient so that a coupler built in this way should be very broadband, indeed. The conclusion is an unexpected one, since we know that a uniform coupler having unequal phase constants can never give complete power transfer.

Nevertheless, the principle may be demonstrated using a pair of coupled pendulums whose lengths are continuously varied so that the one which is initially shorter finally becomes the longer of the two. In Fig. 1 is shown a typical result when the longer pendulum is initially excited. Fig. 1(a) shows how the periods (τ) and the coupling (k) vary with time. There is a fluctuation of energy which is quite small at first, but which increases until at the time the periods become equal, the energy is equally divided and the pendulums are in phase. (When the shorter pendulum is initially excited, they are 180° out of phase.) With increasing time the energy is finally transferred to the other pendulum with small residual fluctuations which gradually diminish. It appears then that, while the transfer of energy is almost complete, it will not be complete unless the difference between the periods approaches infinity at the beginning and end of the process.



(a) (b)

Fig. 1—Transfer of energy from one to another of two coupled pendulums whose periods are varying with time.

In looking for applications for Cook's coupling scheme the writer discovered that by varying both the coupling coefficient and the phase constants of the waveguides simultaneously, the residual power fluctuations may be substantially eliminated. The design of such a coupler may appear to be complicated, but actually the requirements are very simply expressed by a new principle of broad-band coupling which is here called "normal mode warping." Using this principle it should be possible to build wave couplers providing any desired degree of power division over very large bandwidths, limited only by the bandwidth capacity of the waveguides themselves. It may be applied equally well to nonreciprocal and reciprocal structures, and in a wide variety of ways. It is the purpose of this paper to explain in terms which are nonmathematical how normal mode warping can be employed.

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[1] S. E. Miller, "Coupled wave theory and waveguide applications," *Bell Sys. Tech. Jour.*, vol. 33, p. 661; May, 1954. See this paper for additional bibliography on wave coupling.

[2] B. M. Oliver, "Directional electromagnetic couplers," PROC. IRE, vol. 42, p. 1686; November, 1954.

Twisted Waveguide as Prototype Broad-band Coupler

Perhaps the easiest way to explain the principle of mode warping is to begin with a twisted bi-refringent medium. Fig. 2 shows in cross section a long dielectric strip which is twisting in a clockwise direction with propagation into the paper. We may think of this as being either an unshielded dielectric waveguide, or as a dielectric fin inside of a circular waveguide sheath. In either case, we know from experiment that if we launch a linearly polarized wave with its electric polarization either parallel or perpendicular to the dielectric fin, it will propagate along the twist with its polarization remaining, to a first approximation, either parallel or perpendicular to the fin at all points. Thus, the polarization will rotate with rotation of the fin. We know experimentally that if the twist is performed too rapidly, some depolarization will result. On the other hand, if the twist is long and gradual, the polarization will remain quite linear at all points.

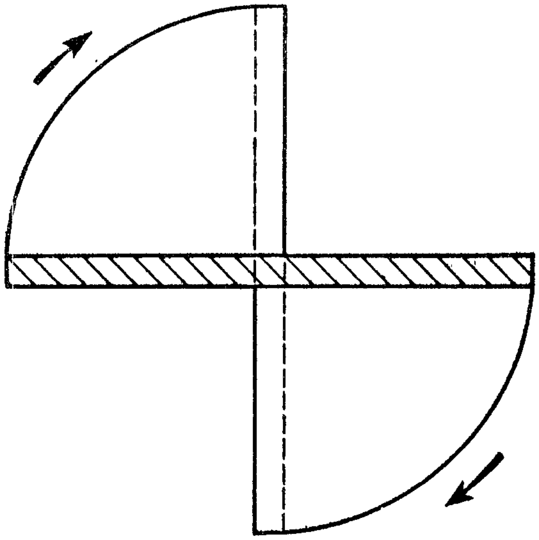

Fig. 2—End view of a twisted dielectric waveguide.

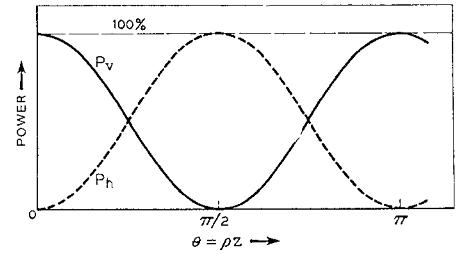

Fig. 3—Power transfer between vertical and horizontal polarizations in a twisted birefringent medium.

In Fig. 3 is shown the way in which power in the vertical and horizontal polarizations varies with distance along the twist. If the twist in the waveguide is just 90°, as suggested in Fig. 2, a vertically polarized wave introduced at one end will emerge as a horizontally polarized wave at the other end. Thus, we find that the twist section, by some coupling mechanism not yet defined, can transfer 100 per cent of the power in a vertically polarized wave to a horizontally polarized wave. Moreover, we know that this transfer is not frequency sensitive, and as shown in Fig. 3, it occurs smoothly and without the ripples indicated in Fig. 1. We suspect, therefore, that the twist represents a preferred way of effecting a 100 per cent power transfer between the two modes. Let us arbitrarily identify the vertically and horizontally polarized modes as the modes between which coupling takes place, and ask how the coupling coefficient and phase constants for these modes vary along the twist.

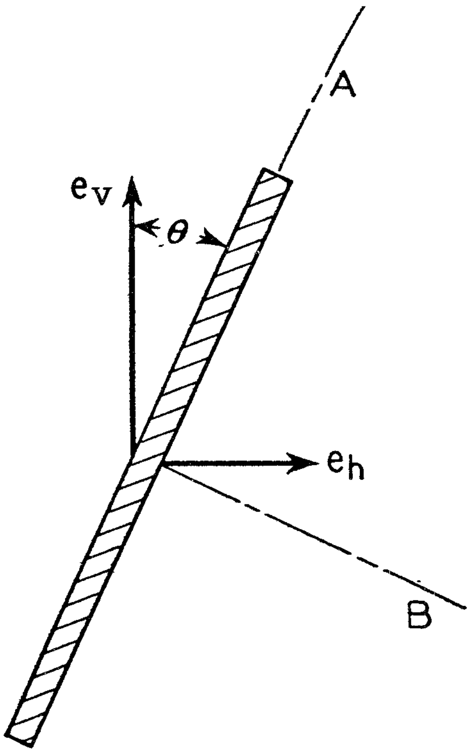

Fig. 4—A uniform birefringent medium producing coupling between vertical and horizontal polarizations.

We will define the phase constants and coupling coefficient at any point along a twist as being the same as those of a uniform (nontwisted) dielectric strip having the same cross-sectional geometry. By rather straightforward analysis which will not be included here, we can determine these parameters for the uniform birefringent waveguide indicated schematically in Fig. 4. The results are shown in Fig. 5. (β_a and β_b are the phase constants for waves polarized along the A and B axes of Fig. 4).

We see that when $\theta = 0$ and when $\theta = \pi/2$, there is a maximum difference between the phase constants β_v and β_h. Moreover, there is no coupling between the polarizations, as we can verify, since we know that a vertically or horizontally polarized wave will travel indefinitely along such a uniform waveguide without depolarization. When the strip is oriented at 45° ($\theta = \pi/4$)

the phase constants for vertical and horizontal polarizations are equal, but now we have a maximum of coupling. Again, we can verify this, since a vertically polarized wave will be converted into a horizontally polarized wave after traveling through a certain length of such a medium. Now if we let θ be a function of distance Z in the direction of propagation, we can use Fig. 5 to describe the parameters for a gradual twist. Finally, we predict that if we can build any pair of waveguides (or transmission modes) with the variation of coupling coefficient and phase constants shown in Fig. 5, we should obtain power transfer properties as shown in Fig. 3. θ is now simply a parameter relating k, β_1, and β_2 as a function of length, but it is still convenient to think of it as the angle of the equivalent twist section. Fig. 3 shows that if the coupler is made of length $\theta=\pi/2$, a complete power transfer will take place. A 3-db coupler will be provided by length $\theta=\pi/4$; and, in fact, any desired division is obtainable by using the proper length. All of these should be broad-band, because we know that the twist prototype is broad-band.

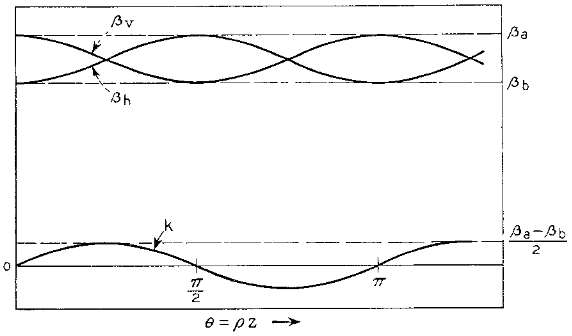

Fig. 5—Coupling coefficient and phase constants for vertical and horizontal polarizations in a twisted birefringent medium.

It might appear that since the desired power transfer is obtained for only a particular length, it would therefore be as frequency-sensitive as matched velocity couplers. Actually, the electrical length is not important. It is simpler to think in terms of the twist prototype, where the only requirement for the desired transfer is that the total twist angle θ be chosen correctly.

One way in which these design requirements may be met is suggested in Fig. 6 for a pair of coupled rectangular waveguides providing complete power transfer ($\theta=\pi/2$). The top wall has been removed to show a divided aperture tapered so as to give a coupling coefficient which varies as the sine of the distance from one end. At the same time the dividing partition is warped so as to produce the cosinusoidal crossover of phase constants. The phase constants could also be adjusted by the insertion of a variable amount of dielectric loading. The vertical vectors represent the square of the electric field present in the two waveguides at various cross sections along the structure when all of the power is initially inserted at (a). Complete transfer takes place with all the field appearing at (d) and none at (b).

The phase relations between the field vectors on the two sides of the coupling aperture are of interest. We know that in the case of a coupler employing uniform waveguides, the induced wave in one waveguide is always 90 degrees out of phase with the driving wave in the other waveguide. This is also true for the two coupled modes (e_v and e_h) in a uniform birefringent medium. However, we know that if we launch a linearly polarized wave on the twist medium with polarization parallel to one of the principal cross-sectional axes, then the wave will remain linearly polarized. Consequently, for this medium the vertically and horizontally polarized modes will have a zero or 180°-phase relation at all points along the medium. It follows that the coupler of Fig. 6 which was derived from the twist medium must also have a zero or 180 degree phase relation between the field components on opposite sides of the coupling partition at every cross section. This situation is illustrated in Fig. 7, where the transverse electric field is plotted for a series of cross sections of the coupler. The input end is shown at the top, and the output end at the bottom. The left-hand column represents the field configurations when the wave is initially launched in the smaller of the two waveguides. The right-hand column is for a wave launched in the larger of the two waveguides.

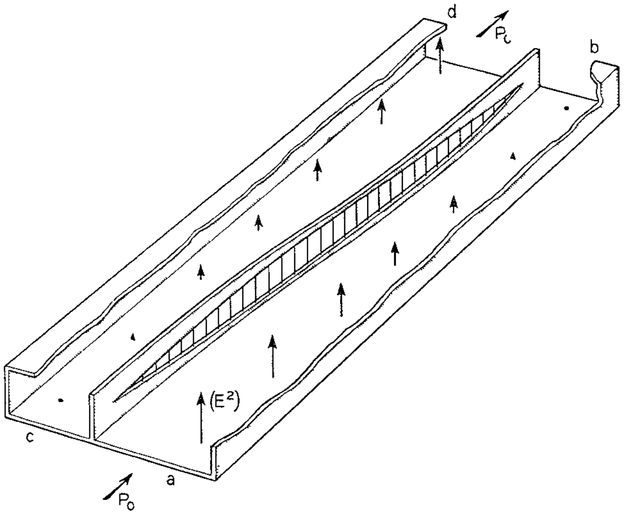

Fig. 6—A broad-band 100 per cent power transfer coupler using mode warping.

It may be seen that, when the wave is launched in the larger waveguide terminal, it emerges at the other end of the coupler from the larger waveguide terminal. Moreover, at the center cross section where the two waveguides have the same phase constant, the energy is equally divided, and we recognize this as the even symmetric normal mode for the local cross section. In fact, the wave travels throughout the length of the coupler in the local normal mode which has the higher phase constant.

Conversely, if the wave is launched in one of the smaller waveguide terminals, it appears at the center cross section in the odd symmetric mode and emerges from the smaller waveguide at the far end. Thus it travels throughout the coupler in the local normal mode having the lower phase constant.

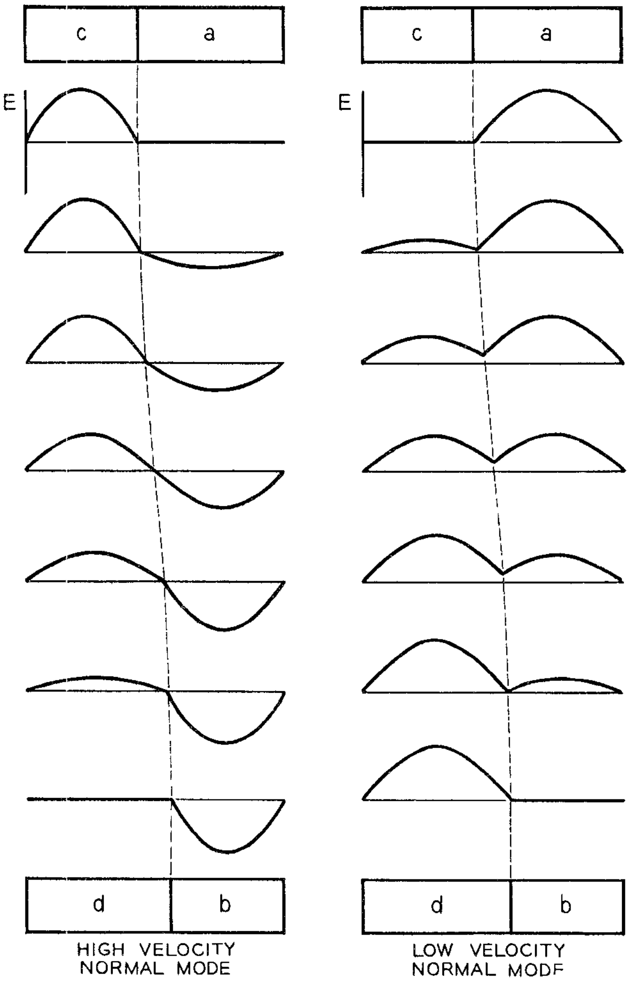

Fig. 7—Warping of odd and even symmetric modes in the coupler of Fig. 6.

Principle of Broad-Band Coupling by Normal Mode Warping

We have discussed a twisted birefringent medium and a rectangular waveguide coupler, both of which are examples of normal mode warping. We will now attempt a statement of the principle which is basic to all such couplers. We assume a waveguide system having two modes of propagation which are to be coupled so as to effect transfer of power. There are, then, two normal modes for this system, which are the dual of the coupled modes, and which may vary in field pattern from point to point along the structure, depending upon the phase constants and coupling coefficients of the coupled modes. Provided these parameters vary slowly and smoothly along the structure, then if all of the power is injected in one of the normal modes at one end, it will remain in one of the normal modes at successive cross sections and will emerge in one of the normal modes at the far end. This situation will be independent of frequency.

On the other hand, if both normal modes are excited at one end, power will be transmitted through the structure in both normal modes and emerge in both modes. This is what happens in conventional matched β couplers, and interference between the modes can vary as frequency changes.

The objective of a broad-band design should then be: (1) to adjust parameters at the ends of the structure so that the normal modes at those points are identical with the desired input and output field excitations; and (2) to vary smoothly the parameters along the structure so that the normal modes are transmuted or "warped" from the one to the other set of field excitations. In this way the power will at all points be in one of the normal modes only, and interference between modes is avoided.

Examples of Mode Warping

We have already seen how this objective was achieved in the birefringent twist. Power was injected in one of the normal modes. This mode was smoothly warped from vertical polarization at one end to horizontal polarization at the other end by twisting. If we had chosen to excite this medium with a wave polarized at 45° to the birefringent axes, both of the normal modes of the medium would have been excited equally. As a result, equal amounts of power would have propagated down the twist in the two normal modes and would have arrived at the far end with relative phases which would depend upon the phase velocities of the two modes and the total distance traveled along the twist. The output polarization in general, would be elliptical and would be frequency dependent.

In the case of the coupled waveguides of Fig. 6, the normal modes at the ends corresponded to dominant wave excitation of the separate waveguide terminals, and they were smoothly warped from one set of terminals to the other. Excitation of one of the waveguide terminals would cause power to flow through the system in only one of the normal modes. If the waveguide structure had been cut at the center, either half of it would constitute a 3-db coupler. With excitation of one of the end terminals, the power at the center cross section will exist entirely in one of the local normal modes, which requires equal voltages in the two waveguides. Thus, the 3-db power division should be very broad-band. On the other hand, if both the waveguide terminals at one end were excited, power would flow in both of the normal modes. The relative phases of these modes arriving at the center would vary with frequency, and hence the power division would be frequency dependent.

In Fig. 8 is shown another example of mode warping, where ferrite is employed in rectangular waveguide to produce a broad-band circulator. Two rectangular waveguides are coupled by a long divided aperture. At the left hand end, both waveguides are occupied by thin tapered slabs of ferrite located off center in the waveguide cross section. These terminate in knife edges at the center where the coupling aperture is largest. From here on, the dividing partition is deflected so as to alter the phase constants of the two waveguides. We may identify four distinct regions in this coupler. The regions between (*a*) and (*b*), and between (*d*) and (*e*) are for the purpose of matching to the waveguide terminals.

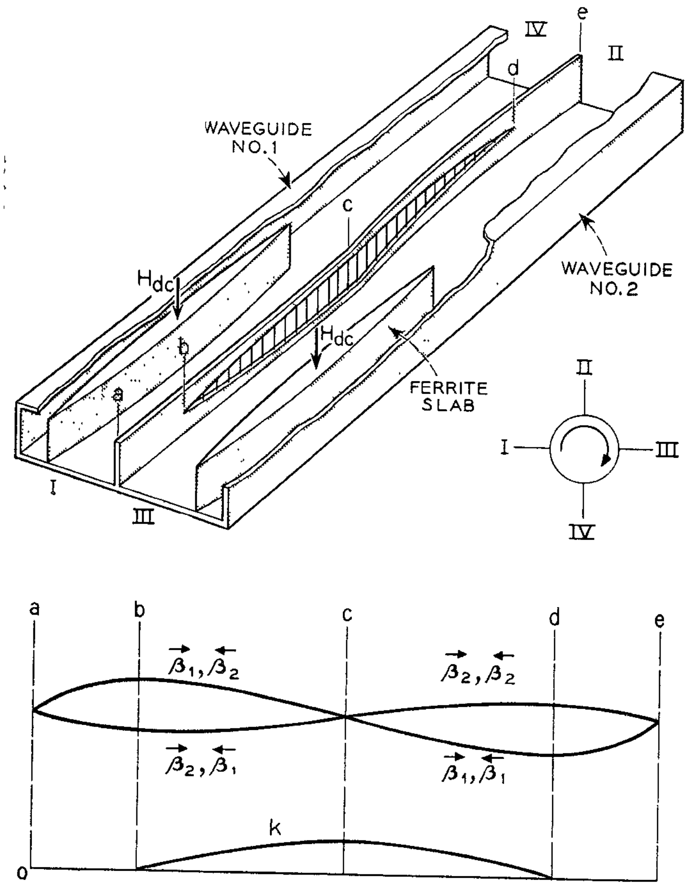

Fig. 8—Rectangular waveguide circulator employing mode warping.

In the regions between (*b*) and (*c*), and between (*c*) and (*d*) mode warping occurs. Both of these regions are equivalent to 45-degree twists of the birefringent prototype, and power in either of their input terminals will be equally divided between the two waveguides at the center cross section (*c*). Thus they operate as broadband hybrids. Section (*cd*) is like one-half of the structure of Fig. 6, and it operates in the same manner. Section (*bc*), on the other hand, operates like a nonreciprocal hybrid, and by virtue of its transversely magnetized ferrite slabs it has nonreciprocal phase constants as shown at the bottom of the illustration. For propagation fron left to right, the ferrite in waveguide 1 exhibits a permeability greater than one, while the ferrite in waveguide 2 exhibits a premeability less than one. For propagation from right to left, the situation is reversed. We may therefore analyze the behavior in the following manner.

A wave entering at terminal *I* will, by the time it arrives at cross section (*b*), be traveling in the normal mode having the higher phase constant. As it passes through section (*bc*) the mode will be warped so that at (*c*), the power will be equally divided between the two waveguides in the even symmetric mode, which has the higher phase constant. Mode warping will continue through section (*cd*), and all of the power will be transferred to waveguide 2, which has the higher phase constant at section (*d*). Thus, power entering at *I* will be delivered to *II*.

A wave entering at *II* will return to cross section (*c*) with power equally divided in the even symmetric mode. From this point on, however, the situation is changed because of the nonreciprocal behavior of the ferrite. Now, waveguide 2 will have a phase constant which is higher than waveguide 1, and the wave which is traveling in the higher phase constant mode will be delivered to terminal *III*. The circulation order of the terminals is therefore as shown by the circulator symbol at the right of the waveguide.

In this structure we note that the average phase constant is not necessarily constant. Nevertheless, the coupling coefficient and phase constant difference are varied sinusoidally as shown in Fig. 5.

CONCLUSION

It has been shown that Cook's scheme for producing broad-band directional couplers by variation of the phase constants may be generalized by simultaneously varying the coupling coefficient. For the simplest case, the difference between the phase constants should vary cosinusoidally and the coupling coefficient should vary sinusoidally with distance along the coupler. Such a programming of the coupling parameters corresponds directly to a twisted birefringent medium (such as a metal waveguide having a flattened cross section) where the rate of twist is constant. Since this medium is easy to analyze and to visualize physically, it has been used as a prototype for the design of several different types of couplers, all of which are based on the same principle. This principle, which in retrospect sounds rather obvious, is simply that in order to avoid interference effects between two modes of propagation in a multimode waveguide system, we should avoid exciting more than one of the normal modes. Also, by gently warping the waveguide structure, it is possible to warp the field configuration of the desired normal mode so that it will produce the required power division at the terminals of the system without appreciably scattering power into other unwanted modes.

By avoiding wave interference, such couplers should in principle, be dependent of frequency. However, the requirement that warping be smooth and gradual also dictates that these couplers must be many wavelengths long. It is possible that they will be most useful in the millimeter wavelength range, where such electrical lengths are physically short.